Department of Higher Education
University of Computer Studies, Yangon
Fifth Year (B.C.Sc. / B.C.Tech.)
Re - Examination
Mathematics of Computing V (CST-501)
September, 2018

Answer ALL questions. **Time allowed : 3 hours.**

1(a) Given each of the following (one-step) transition matrices of Markov chain, draw the state transition diagram and determine the class of the Markov chain and whether they are recurrent.

$$P = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

(b) Consider the following blood inventory problem facing a hospital. There is need for a rare blood type, namely, type AB, Rh negative blood. The demand D (in pint) over any 3-period is given by P{D=0}=0.4, P{D=1}=0.3, P{D=2}=0.2 and P{D=3}=0.1. Suppose that there are 3 days between deliveries. The hospital proposes a policy of receiving 1 pint at each delivery and using the oldest blood first. If more blood is required than is on hand, an expansive emergency delivery is made. Blood is discarded if it is still on the shelf after 21 days. Denote the state of the system as the number of pints on hand just after a delivery. Thus, because of the discarding policy, the largest possible state is 7.

Construct the (one-step) transition matrix for this Markov chain.

2(a) What are the steady-state probabilities of the following Markov chain?

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.2 & 0.75 & 0.05 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

(b) A video cassette recorder manufacturer is so certain of its quality control that it is offering a complete replacement warranty if a recorder fails within 2 years. Based upon compiled data, the company has noted that only 1 percent of its recorders fail during the first year, whereas 5 percent of the recorders that survive the first year will fail during the second year. The warranty does not cover replacement recorders.

(i) Formulate the evolution of the status of a recorder as a Markov chain whose states include two absorption states that involve needing to honor the warranty or having the recorder survive the warranty period. Then construct the (one-step) transition matrix.

(ii) Find the probability that the manufacturer will have to honor the warranty.

3(a) Midtown Bank always has two tellers on duty. Customers arrive to receive service from a teller at a mean rate of 40 per hour. A teller requires an average of 2 minutes to serve a customer. When both tellers are busy, an arriving customer joins a single line to wait for service. Experience has shown that customers wait in line an average of 1 minute before service begins. Determine W_q, W, L_q, and L for this queueing system.

(b) Suppose that a queueing system has two servers, an exponential interarrival time distribution with a mean of 2 hours, and an exponential service-time distribution with a mean of 2 hours for each server. Furthermore, a customer has just arrived at 12:00 noon.

(i) Suppose that no additional customers arrive before 1:00 P.M. Now what is the probability that the next arrival will come between 1:00 and 2:00 P.M.?

(ii) What is the probability that the number of arrivals between 1:00 and 2:00 P.M. will be 2 or more?

4(a) A Grocery Store has a single checkout stand with a full-time cashier. Customers arrive randomly at the stand at a mean rate of 20 per hour. The service-time distribution is exponential, with a mean of 2 minutes. This situation has resulted in occasional long lines and complaints from customers. Therefore, because there is no room for a second checkout stand, the manager is considering the alternative of hiring another person to help the cashier by bagging the groceries. This help would reduce the expected time required to process a customer to 1.5 minutes, but the distribution still would be exponential. The manager would like to have the percentage of time that there are more than two customers at the checkout stand down below 25 percent. She also would like to have no more than 5 percent of the customers needing to wait at least 5 minutes before beginning service, or at least 7 minutes before finishing service.

Use the formula for the M/M/1 model to calculate L, W, W_q, L_q, P_0, P_1, and P_2 for the current mode of operation. What is the probability of having more than two customers at the checkout stand?

(b) Consider a telephone system with three lines. Calls arrive according to a Poisson process at a mean rate of 6 per hour. The duration of each call has an exponential distribution with a mean of 15 minutes. If all lines are busy, calls will be put on hold until a line becomes available. Calculate P_n, L, W, W_q and L_q for this queueing system (with t =1 hour and t =0, respectively, for the two waiting time probabilities).

5(a) Apply the inverse transformation method as indicated below to generate three random observations from the uniform distribution between -10 and 40 by using the following uniform random numbers: 0.0965, 0.5692, 0.6658.

(i) Apply this method graphically. (ii) Apply this method algebraically.

(b) Generate three random observations from the following probability distribution.

The distribution whose probability density function is

$$f(x) = \begin{cases} \frac{1}{200}(x-40) & if 40 \le x \le 60 \\ 0 & Otherwise \end{cases}$$

Use the uniform random numbers:

0.096, 0.569, 0.665, 0.764, 0.842, 0.492, 0.224, 0.950, 0.610, 0.145

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